Quantum Computers

Investigating Uncertainty

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Abstract

This research report aims to investigate methods employed in creating a quantum computer. By starting with a look at the fundamental building blocks (qubits) and progressing onto making and retaining qubits, the report gives information and comparisons on alternative methods, and concludes with practical uses.

Contents

Introduction

Classical computers have been decreasing in size according to Moore's Law since 1965; the law states that that the number of transistors (basic components of modern computational electronics) on a chip doubles every 18 months[1]. With the technology progressing at this rate for many years it became apparent that the transistors would soon be only a few atoms in diameter. This presented a problem to the scientists; on such a small scale the circuit would follow the laws of quantum physics, not of classical physics. By 1985 it was shown that a device which followed these quantum laws could help solve a set of infinitely hard mathematical problems, and soon a full-scale development was underway to build a so-called 'Quantum Computer'[2].

Due to the 'new' physics involved in building a quantum computer there have been many devices needed to be specifically engineered to work around the laws of quantum physics. By stripping these components to the bare minimum this article will explain how quantum computers work (with relation to classical computers), what devices are involved in a quantum computer, and will finally touch on future developments in the field.

Qubits

A classical computer is, in its simplest form, a collection of ones and zeroes; these ‘bits’ can be anything from pulses of light, to differences in voltage [fig. 1.], to differences in magnetic tape[3].

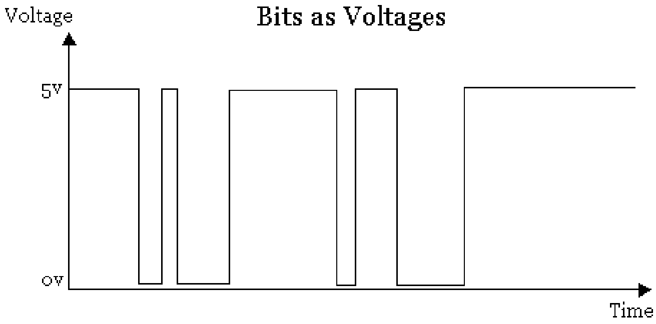

Fig. 1.
Bits as Voltages

A quantum computer is similarly composed of quantum bits, or ‘qubits’, which must be microscopic particles to harness quantum effects. Qubits, expressed as vectors, can be one, $|1\rangle$, or zero, $|0\rangle$, as a classical bit. As the tiny particles are quantum in nature they can display a superposition of both, that is one and zero at the same time. This can be understood by considering the demonstration of the movement of two different waves along one string; the point of intersection produces one combined wave of collective amplitude [fig. 2]. The combined wave is both waves at the same time.

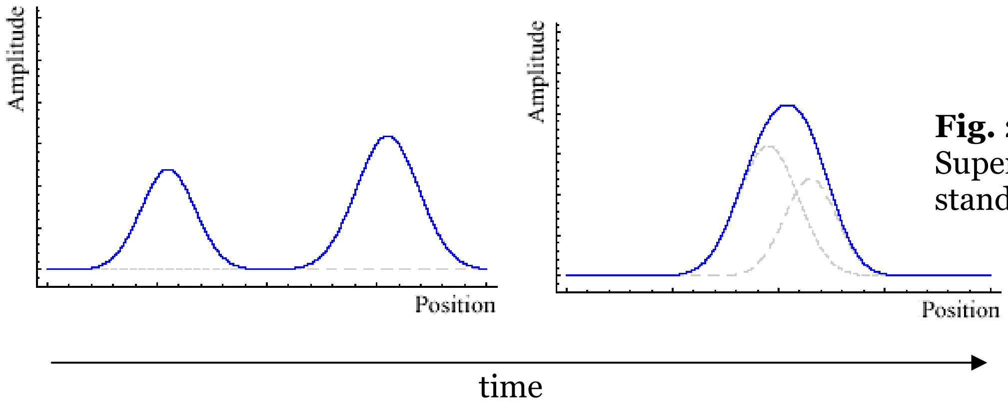

Fig. 2.
Superposition of standing waves

The qubit is then in a combination of the states, represented as:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \qquad (1)$$

α and β are probability functions where $|\alpha|^2 + |\beta|^2 = 1$

The combination of the two baseline states results in the ‘entangled state’, which gives rise to the power of quantum computers (explained in detail later).

The most common qubits are single electrons, ionised alkaline earth metals or photons. If ionised Calcium, $^{40}Ca^+$, is considered as a qubit:

Ground state = $1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 4s^1$

The lone outer-shell electron ($4s^1$) allows the standard bit functions of the qubit [fig. 3]. The one and zero positions are given by the intrinsically quantum nature of the spin of the outer-shell electron[4].

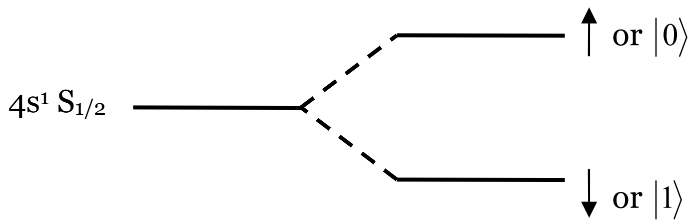

Fig. 3.
The ‘on’ and ‘off’ states of a $^{40}Ca^+$ qubit

A photon (and hence other quantised particles) can be proved to be in two states at once, by accepting that light travels as a particle (a photon), and by repeating Young’s Double Slit experiment. Using a lamp dimmed to emit a single photon at a time, when a considerable gap is left between each emission, the exposure on the screen over a period of time produces the same interference pattern. This result implies that the photon interferes with itself [fig. 4].

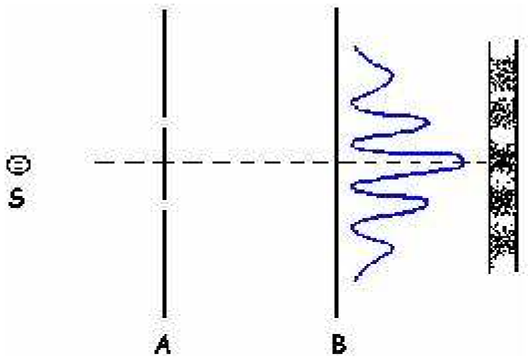

Fig. 4.
Double slit interference from single photons

The diffraction pattern produced is described by equation 2:

$$n\lambda = \frac{xd}{L} \qquad (2)$$

Where: λ = wavelength, x = fringe distance, d = slit separation, L = slit-to-screen distance

To produce a clearly visible set of fringes with a separation of 1cm, with red light (λ = 650nm), through slits separated by 0.5mm, the screen must be:

$$650 \times 10^{-9} = \frac{1 \times 10^{-2} \times 5 \times 10^{-4}}{L}$$

$$L = 7.69m$$

Although it is known that the photon leaves the lamp, and the photon arrives at the screen, the actual path that the photon takes is not known, so it is said to travel through both slits at the same time[6]. Superposition is a way of describing an object when some information is not known about the object. In the case of the qubits it is not known in which direction the electron is spinning, hence whether the qubit currently represents a one or a zero. It is therefore said to represent both. Once the state is measured however, the superposition collapses, and the qubit takes one of the standard states, one or zero.

Turning an Ion into a Qubit

The first step on the road to building a quantum computer is to produce an array of qubits which can be used in further calculations.

At regular temperatures any material is in excited energy states, and is constantly switching between energy levels as it absorb and emit energy. This is no good for a device expected to retain data! To remove the volatile nature of the qubits, they need to be lowered to their natural ground states, i.e. an energy state where they do not jump between energy levels. Cooling removes the energy from the molecules because equation 3 is obeyed:

$$KE_{molecular} = \tfrac{3}{2}kT \tag{3}$$

The average translational kinetic energy, *KE*, of the molecule is directly proportional to the temperature in Kelvin, *T*, where the constant is three halves of Boltzmann's Constant, *k*[7]. When the temperature is lowered, the internal energy of the system decreases, so the particles are more stable and jump between energy levels less. Cooling in liquid nitrogen reaches temperatures of about 63K[8]; cooling in liquid helium reaches temperatures of about 1K[9].

KE at room temperature 298K :

$$\begin{aligned} KE_{molecular} &= \tfrac{3}{2} \times 1.38 \times 10^{-23} \times 298 \\ &= 6 \times 10^{-21} J \end{aligned}$$

KE when cooled in liquid nitrogen:

$$\begin{aligned} KE_{molecular} &= \tfrac{3}{2} \times 1.38 \times 10^{-23} \times 63 \\ &= 1.3 \times 10^{-21} J \end{aligned}$$

KE when cooled in liquid helium:

$$\begin{aligned} KE_{molecular} &= \tfrac{3}{2} \times 1.38 \times 10^{-23} \times 1 \\ &= 2.07 \times 10^{-23} J \end{aligned}$$

All of these show the average kinetic energy per molecule; as the temperature decreases, the average energy of the molecules also decreases. Using the mass of a $^{40}Ca^{+}$, 6.655×10^{-26} kg, the average speed of the molecules can be determined.

$$KE = \tfrac{1}{2}mv^2 \qquad (4)$$

$$KE = \tfrac{1}{2} \times 6.655 \times 10^{-26} \times v^2$$

Speed at room temperature 298K :

$$6 \times 10^{-21} = \tfrac{1}{2} \times 6.655 \times 10^{-26} \times v^2$$

$$v = 424ms^{-1}$$

None of these methods is effective in forcing the qubits into their ground states, so a more dramatic process is required: laser cooling. Laser cooling reaches temperatures of less than one ten thousandth of a Kelvin.
Laser Cooling:

$$\begin{aligned} KE_{molecular} &= \tfrac{3}{2}kT \\ &= \tfrac{3}{2} \times 1.38 \times 10^{-23} \times 0.0001 \\ &= 2.07 \times 10^{-27} J \end{aligned}$$

$$KE = \tfrac{1}{2}mv^2$$

$$2.07 \times 10^{-27} = \tfrac{1}{2} \times 6.655 \times 10^{-26} \times v^2$$

$$v = 0.25ms^{-1}$$

The speed of the particle is radically reduced at the lower temperature, so it is apparent that the lowest temperature possible must be reached to slow the particle down sufficiently in order to control the state of the qubit; to remove the volatile nature.

A laser is a source of monochromatic, in-phase, coherent light. This means that the photons are of the same wavelength, they are all travelling in the same direction, and at the same speed. These properties allow the beam to be highly tuneable, allowing ultra fine adjustments to be made. The light is produced initially by passing a current through a gas, which causes a fluorescent effect. This energy is soon emitted from the gas molecules as light. If the new photons collide with an already excited gas particle, the particle emits a photon of light of exactly the same wavelength, travelling in exactly the same direction. Mirrors are used to further amplify the light[10].

Laser cooling works in a similar way to conventional evaporation; evaporation occurs when some energy is given to some molecules (e.g. of water), which escape, leaving the less energetic ones behind. The internal energy of the water is reduced, so its overall temperature is reduced, despite energy being previously given to the system[11]. Laser cooling works by bombarding the particle with photons of light, which 'knocks off' the energy, leaving the system cooler. This process requires the correct frequency of light, and is governed by equation 5.

$$E = hf \tag{5}$$

The energy, E, supplied by a photon of light is directly proportional to the frequency, f, of the light, where the constant is Planck's Constant, h.

Some complications are introduced during cooling; as the molecules cool, they require less and less energy to knock the electron from the excited state. As the system is quantised arbitrary values of energy can not be given to the molecules, so the frequency of the photon must be reduced at a rate proportional to the cooling of the molecule.

Holding Qubits in Place

Now that the qubits have been prepared in a ground energy state, they must be held away from everything else, so as not to take them out of the ground state which would happen if they gained energy from collisions. There are two emerging technologies which each deal with the problem in different ways; both require a vacuum: the simplest method to avoid collisions. The first is an electromagnetic ion trap, comprised of four electromagnetic rods with two retaining end rods [fig. 5].

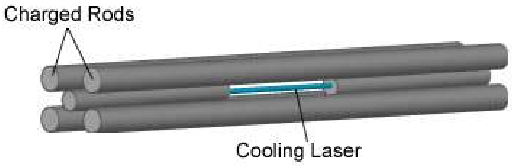

Fig. 5.
An Ion Trap

The four outer rods have an alternating current sent through them; the two retaining end rods (consisting also of the laser) have the opposite alternating current through them. As the current passes through the outer rods an electromagnetic field is set up. The $^{40}Ca^+$ ions are placed in the middle of the apparatus, in the cooling laser. The magnetic fields of the rods combine and cancel out along the central meridian[12] (blue dashed line) [fig. 6]. The ions sit in this central field, not moving, as there is no resultant force. They are held away from all other external forces.

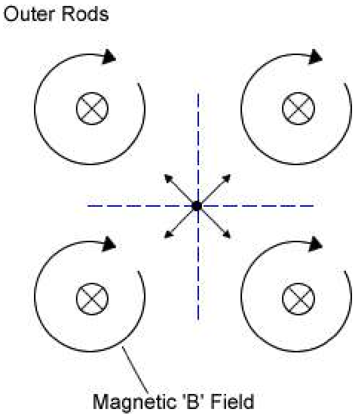

Fig. 6.
The current is shown entering the page. The magnetic fields combine to produce no resultant force; the ion is held in place.

If the ion tries to drift away from the central meridian the combined magnetic forces are unbalanced. The moving ion then interacts with the unbalanced magnetic field, according to Fleming's Left Hand Rule [fig. 7], and this causes movement. It is crucial that the ion is moving in the opposite direction to the current for it to be drawn back into the centre of the rods [fig. 8]. The retaining end rods fulfil this purpose. They are π out of phase with the four outer rods, which results in the ion always moving in the opposite direction to the current.

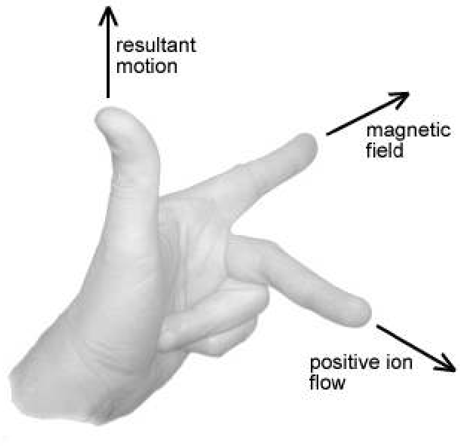

Fig. 7.
Fleming's Left Hand Rule[13]

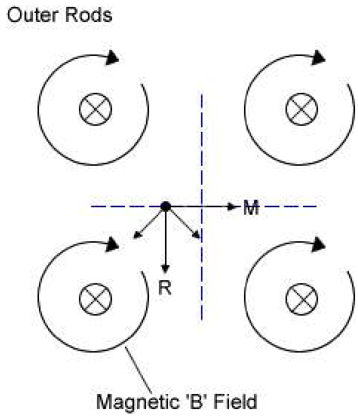

Fig. 8.
The resultant magnetic field (R) if the ion is not in the centre of the electromagnetic rods.

Now that the ion is not located centrally between the electromagnetic rods there is a resultant force. If Fleming's Left Hand Rule is applied, the first finger points along the line R, ' the resultant magnetic field', the second finger, 'positive ion flow', is pointing out of the page which means that the thumb, 'resultant motion' (M), shows the of the ion is moved back towards the central meridian (blue dashed line).

The second method is to use lasers as an optical lattice. Two or more lasers are aimed together at right angles, resulting in a similar interference pattern shown in the Young's Double Slit experiment [fig. 4]. The qubits, due to their low energy state, fall into the energetic minima of the pattern [fig. 9].

The lasers are best thought of as a stream of ping pong balls (photons). The qubits should be thought of as tennis balls. The ping pong balls should be fired at the tennis balls, which will be pushed (slowly) around, until the tennis balls have been driven into the gaps

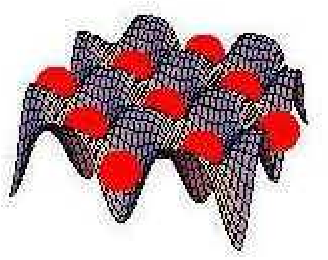

Fig.9.
An Optical Lattice[14]

between streams of ping pong balls. The tennis balls will sit in the 'dark areas', and will not force themselves into the stream of ping pong balls again.

Both methods of holding the qubits in place seem to work equally well, but are both fairly intricate to set up and maintain. The electromagnetic ion trap would allow better visibility of the ions (for experimentation purposes) as they permanently sit in a laser beam, which would scatter around them, producing visible results [fig. 10].

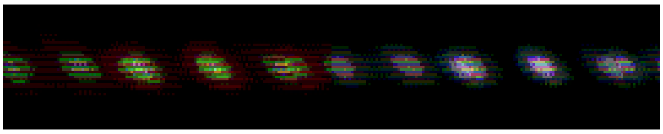

Fig. 10.
A false colour photograph taken with a monochrome digital camera, it shows light scattering off the ions[15].

The ions in the optical lattice only cause scattering of light as they are cooled. Once the ions are in the dark areas of the lattice they will be invisible. The electromagnetic ion trap may also be more useful in creating quantum logic gates, as to ensure interaction between ions the lasers in the optical lattice need to be momentarily switched off and back on again – this is clearly a hazardous system, jeopardising data integrity.

Obtaining Answers

With an array of useable qubits prepared and held in the correct state it is possible to perform simple calculations, which will be the basis of subsequent logic gates, and hence a fully scalable quantum computer.

For a classical computer to store data, the register can only store one number at a time. For instance, in a register of 3 bits there are $2^3 = 8$ combinations:

111 = 7
010 = 2

A quantum register (where a qubit is represented as a vector ($|1\rangle$ or $|0\rangle$) can also hold individual numbers (represented as tensor products):

$$|1\rangle \otimes |1\rangle \otimes |1\rangle = 7$$

$$|0\rangle \otimes |1\rangle \otimes |0\rangle = 2$$

But once the qubits are in a superposition of states, they can hold many numbers. Here an example superposition is given (remember equation 1):

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

α and β are probability functions where $|\alpha|^2 + |\beta|^2 = 1$

Notice a and b = $\frac{1}{\sqrt{2}}$:

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

$$|\alpha|^2 + |\beta|^2 = 1$$

$$2 \times \left|\frac{1}{\sqrt{2}}\right|^2 = 1$$

If all three of the qubits in this register are put in a superposition of states, the register now stands as:

$$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle \otimes \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle \otimes \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle$$

By expanding (and ignoring the constant) the register holds:

$$|000\rangle + |001\rangle + |010\rangle + |011\rangle + |100\rangle + |101\rangle + |110\rangle + |111\rangle$$

This simple example shows the power of quantum computers. In a three bit register, eight times as much data can be held for each calculation than in a classical register, where the bits can only hold one number[16].

Links

By studying this fascinating new technology of Quantum Computers I have looked at each individual component that is required to create this device, and I have built up a picture of how the device operates. I can see that the links between all of the physics is the Qubit, a positive calcium ion, which is acted on by many different areas of physics.

Electrostatics is used to create the ion, wave/particle superposition theory to understand how it gets its unique quality of taking two states (entanglement) at the same time. Thermodynamics explains how the ion is cooled to keep it virtually stationary, and to keep it stable in a ground state. Electromagnetism and the motor effect is used to keep the qubit in the right place. Once the qubit is in the place where you want it, a laser is used to control it. This is where the maths takes over.

Conclusion

This report has studied the physics of designing and implementing a quantum computer. By searching deep into the nature of the fundamental building block, the qubit, and then investigating the practical nature of taking an atom, turning into an ion by increasing the temperature to 800°C and 'boiling' electrons off to leave it positively charge, cooling it to produce a qubit using laser cooling which is simple to simple evaporation, and finally holding the qubit in place using one of two methods, either an electromagnetic ion trap or a laser optical lattice, I have discovered a great quantity of new physics, covering many different areas of the subject. I found that most of the physics I came across can be explained by and certainly compared to the fundamental level by simpler physics I already know, and by simple thought experiments. Over the last half-century physics and computer science have been progressing at phenomenal rates, with entirely new subject areas being developed. Already scientists are harnessing the newly understood principals of quantum mechanics, and putting them to our advantage. Although a universal quantum computer is still some years away, it has been shown that many of the components required to realise the device have been significantly developed, with many alternatives available.

Motivations

My personal motivations for this research report stems from my fascination of data security. I have a real interest in cryptography, the art of hiding and revealing meanings in masked data, and I realise that with the boom of e-commerce, which relies on hiding credit card numbers as transactions occur on the internet, the most important tool in breaking this security (not for malicious intent, but to create bigger and better security measures) is the factorisation of prime numbers [see Appendix 1 for the importance of prime numbers]. Currently the time for a classical computer to factorise a large number increase exponentially with each digit, models of quantum computers, and early experiments indicate reducing this to a polynomial increase in the time[17].

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Evaluation of Sources

I have used a large range of sources in researching this paper. Some information was taken from school (and undergraduate level [12]) textbooks, which provided useful, readable explanations to some difficult problems. Popular literature ([6]) was also useful in providing clear, understandable clarifications of some areas. It was useful to contact directly a physicist ([4]) working on building a usable quantum computer. Websites and other sources didn't directly contradict each other, but did provide several methods of achieving the same goal; two of these methods were discussed and evaluated on their individual merits earlier. Care was taken when using information on websites (esp. [4]). The data was assessed for validity; sometimes uncertain sites were found to give the most clear and direct explanation which was most useful. Melting points and other chemical data ([8],[9]) was obtained from a chemical data site, but the information was double checked on a second site. Some sources ([7],[17]) were particularly helpful and reliable. They were clearly written by knowledgeable people who work with the ideas explained on a day to day basis, and they give clear and detailed explanations.

Appendix 1

The Maths of RSA

Extracted mainly from *The Code Book*, by Simon Singh…

1. Alice (the intended recipient) picks two giant prime numbers, p and q, which she must keep secret.
2. She multiplies these numbers together to get N, and picks another number e.
    - e and $(p-1)(q-1)$ should be relatively prime
3. Alice publishes e and N in a cryptographic telephone directory.
4. Bob (the sender) prepares the message, M, by encoding it as ASCII binary and encrypting as:

$$C = M^e (\bmod N)$$

5. Exponentials in modular arithmetic are one-way functions, so it is very difficult for Eve (the interceptor) to recover the original message, M.
6. Alice can decipher the message as she has special information, p and q, the prime factors of N, from which she can calculate the decryption key, d:

$$e \times d = 1(\bmod (p-1)(q-1))$$

7. To decrypt the message Alice uses the formula:

$$M = C^d (\bmod N)$$

The security of RSA relies on the inefficiency of factorising large numbers. If Eve were able to find p and q from N, she would have no difficulty in cracking the message, as she knows as much information as Alice.